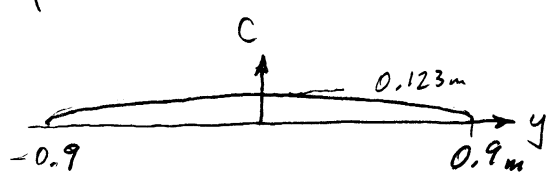
UE Fluids Problem 8 Solution S'08

1a) In level flight, $L = \frac{\pi}{4}\rho V \Gamma_0 b = W$, $\Gamma_0 = \frac{4}{\pi}\frac{W}{\rho V b} = 0.2447$

$$c = \frac{2\Gamma}{Vc_l} = \frac{2\Gamma_0}{Vc_l}\sqrt{1-\left(\frac{2y}{b}\right)^2} = 0.1228\,m\sqrt{1-\left(\frac{2y}{b}\right)^2}$$

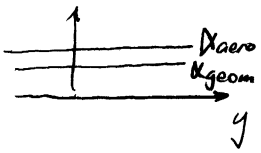
$$\Rightarrow c_0 = 0.1228\,m$$



1b) $c_l = 2\pi\left[\alpha + \alpha_{aero} - \alpha_i\right]$, $\alpha_i = \frac{\Gamma_0}{2bV} = 0.0136\,rad$

$$\alpha_{aero} = \frac{c_l}{2\pi} + \alpha_i = \frac{0.8}{2\pi} + 0.0136 = 0.141\,rad \quad \text{constant}$$

1c) Same camber $\rightarrow \alpha_{L=0} = \phi$, $\alpha_{geom} = \alpha_{aero} + \alpha_{L=0}$ constant offset



1d) $C_{D_i} = \frac{C_L^2}{\pi A R e}$; $C_L = c_l = 0.8$ spanwise constant, $AR = \frac{b^2}{S} = \frac{b^2}{c_0 b\,\pi/4} = 18.66$

$C_{D_i} = 0.0109$ (e = 1 since $\Gamma(y)$ is elliptical)

$C_D = c_d + C_{D_i} = 0.015 + 0.0109 = 0.0259$, $\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.8}{0.0259} = 30.87$

2a) $\Gamma_0 = \frac{4}{\pi}\frac{W}{\rho V b} = \frac{4}{\pi}\frac{3N}{1.2\cdot 12\,m/s\cdot 1.8} = 0.1474$

2b) $\alpha_i = \frac{\Gamma_0}{2bV} = 0.00341\,rad$

$c_l = 2\pi\left[\alpha + \alpha_{aero} - \alpha_i\right]$; $\alpha_{aero} = 0.141$ same airfoil as 1b)

$C_L = \frac{W}{\frac{1}{2}\rho V^2 S} \sim \frac{1}{V^2}$ doubling V decreases C_L by 0.25×, so $C_L = C_l = 0.2$

$\rightarrow \alpha = \frac{C_l}{2\pi} - \alpha_{aero} + \alpha_i = \frac{0.2}{2\pi} - 0.141 + 0.00341 = -0.1058\,rad$

2c) $C_{D_i} = \frac{C_L^2}{\pi AR} = \frac{0.2^2}{\pi\cdot 18.66} = 0.00068$

$C_D = c_d + C_{D_i} = 0.0157$, $\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.2}{0.0157} = 12.75$